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AN ANALYSIS OF THE EFFECTS OF DIFFERENTIAL PHASE FEEDBACK ON A TYPE 2 FEEDBACK CONTROL SYSTEM AS APPLIED TO PHASE-LOCK RECEIVERS

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FOREWORD

This is a technical report of a study conducted by the Electrical Engineering Department of Auburn University under the auspices of the Auburn Research Foundation toward the fulfillment of the requirements prescribed in NASA Contract NAS8-5231. An analysis of a standard phase-lock receiver employing differential phase feedback is presented.

ABSTRACT

A phase lock receiver configuration which consists of a standard phase lock receiver employing differential phase feedback has been proposed for use in the AROD program. The purpose of this configuration is to produce a receiver which has a very stable local oscillator while maintaining an output which tracks the input signal dynamics.

The addition of differential phase feedback to the standard phase lock receiver used in this investigation results in a system which can be analyzed as a Type 2 feedback control system. A mathematical model of the system employing differential phase feedback is obtained by assuming input and output signals and observing the effects of the various operations on the phase of the signals.

An analysis of the system is made using linear feedback control theory. Root locus diagrams of the open loop transfer function are obtained for various values of the system parameters. The initial and final values of the system output and the voltage controlled oscillator output are calculated. The digital computer program utilized to calculate the response of the system to a step input of phase is presented in Appendix A. The calculated response of the system is given in Appendix B.

It is shown that differential phase feedback allows the use of a very stable voltage controlled oscillator while maintaining an acceptable tracking capability.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

I. INTRODUCTION

R. L. Lloyd and H. M. Summer

In recent years, the phase-lock receiver has found wide applications in the field of missile tracking and guidance. One important application is in the field of missile range measurements. Range measurements are made by comparison of transmitted and received signals using techniques similar to those used in radar systems. But unlike radar, the target, which may be either the missile or a ground station, is an active device. The target contains a transponder which receives and retransmits rather than reflecting the signals. The basic component of the transponder is a phase-lock receiver which is used to maintain the received signal and the retransmitted signal phase coherent.

In this system which is a single loop feedback control system, the output of a voltage controlled oscillator (VCO) is compared with the received signal to produce an error signal that is used to correct the frequency of the VCO until the two signals are phase coherent. When the output of the VCO and the received signal are phase coherent, the system is said to be phase locked.

There are many conditions which can cause a phase-lock receiver to have a momentary loss of lock (VCO output and received signal not phase coherent). One such condition is the high-noise environment in which the receivers are normally used. In such an environment, it is desirable to have a stable VCO output to reduce the amount of drift that can occur during the time the system is not in lock.

A narrow band VCO loop will produce a stable VCO but the system will not be able to track rapidly changing input signals. A wide band loop has the tracking capability but lacks a stable VCO. To produce a system which has a good tracking capability and a stable VCO, an additional feedback loop must be added. A system has been proposed which is a standard phase-lock receiver that employs differential phase feedback as the additional feedback loop.

The addition of differential phase feedback to a standard phase-lock receiver results in a system as depicted in Figure 1. Differential phase feedback is employed to produce a composite output signal which tracks the system input dynamics when used in conjunction with a very stable VCO.

If the input signal has an instantaneous and sustained change in phase, the differential phase feedback loop produces a signal which keeps the system in lock until the phase of the VCO can be adjusted to correspond with the input phase. The time constants of the two loops are adjusted such that the differential phase feedback loop is in effect only while the VCO phase lags that of the input.

The purpose of this study is to analyze the system of Figure 1. An analysis is performed which indicates that the desired system performance can be obtained.

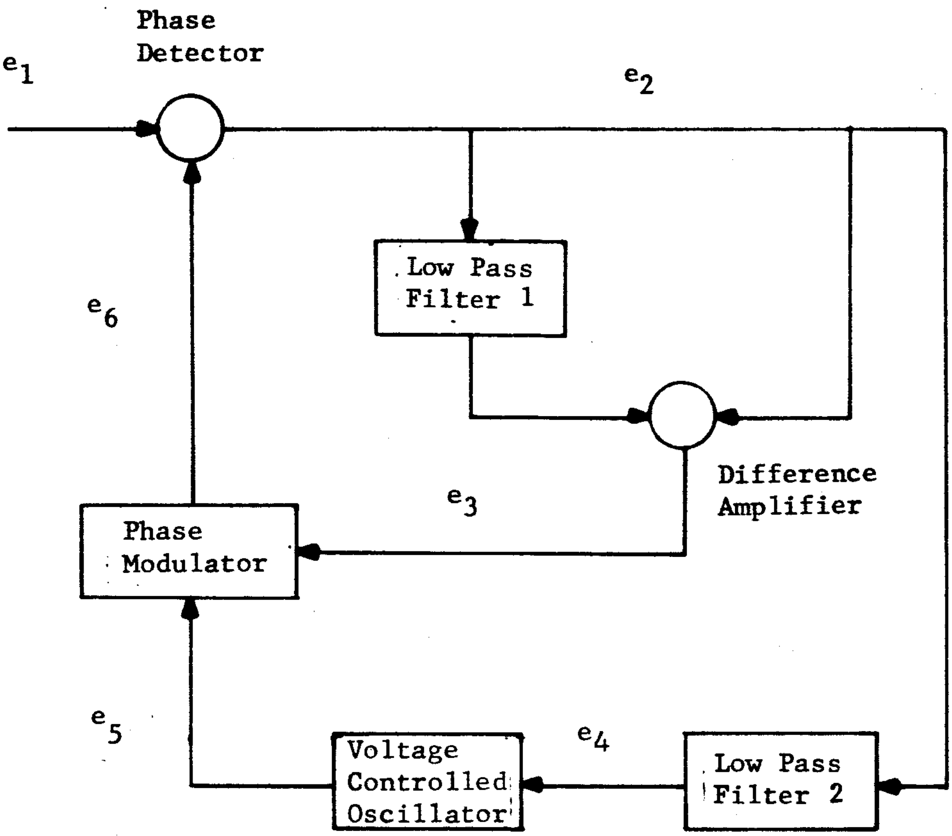


Fig. 1. Block diagram of a phase locked receiver employing differential phase feedback.

II. SYSTEM ANALOGUE

In the analysis of a system employing differential phase feedback, it is desirable to obtain a mathematical model of the physical system represented by Figure 1. This model can be obtained by assuming input and output signals and observing the effects of the various operations upon the phase of the signals. For this analysis, assume that noise free conditions exist and that the input signal is of the form

$$e_1 = E_1 \sin\left[\omega_A t + \phi_A + \theta_{in}(t)\right], \tag{1}$$

where ϕ_A is the phase of the input signal, ω_A is the carrier frequency, and $\theta_{in}(t)$ is a change in the input phase. Also assume that the output signal is of the form

$$e_6 = E_6 \cos\left[\omega_B t + \phi_B + \theta_o(t)\right], \tag{2}$$

where ϕ_B is the phase of the output signal, ω_B is the carrier frequency, and $\theta_o(t)$ is the change in the output phase due to $\theta_{in}(t)$.

A necessary condition for phase lock to occur is that

$$\omega_A = \omega_B. \tag{3}$$

For simplicity, it is assumed that

$$\phi_A = \phi_B . \tag{4}$$

The phase detector output, e_2, is equal to a constant times the difference in the phase of the two input signals. Thus, e_2 is given by

$$e_2 = \frac{E_1E_6}{2}\left[\theta_{in}(t) - \theta_o(t)\right] . \tag{5}$$

In Laplace transform notation, the phase modulator input signal can be represented by

$$E_3(S) = K_1'\left[1 - \tau_{f1}(S)\right]E_2(S) = K_1'\left[1 - \tau_{f1}(S)\right]\frac{E_1E_6}{2}\left[\theta_{in}(S)-\theta_o(S)\right], \tag{6}$$

where K_1' is the gain of the difference amplifier, τ_{f1} is the transfer function of the low pass filter in the differential phase feedback loop and $E_2(S)$ is the Laplace transform of e_2.

In Laplace transform notation, the input to the VCO is of the form

$$E_4(S) = \tau_{f2}(S)E_2(S) = \tau_{f2}(S)\frac{E_1E_6}{2}\left[\theta_{in}(S) - \theta_o(S)\right], \tag{7}$$

where τ_{f2} is the transfer function of the low pass filter in the VCO loop. The output of the VCO can be represented by

$$e_5 = E_5 \cos\left[\omega_B t + \phi_B + \phi_1(t)\right] , \tag{8}$$

where ϕ_1 is the change in phase in the VCO output due to e_4.

The phase modulator output is of the form

$$e_6 = E_6 \cos\left[\omega_B t + \phi_B + \phi_1(t) + \phi_2(t)\right] , \tag{9}$$

where E_6 is equal to a constant times E_5 and ϕ_2 is the change in phase due to e_3.

The value assumed for e_6 must be equal to the value of e_6 given in (9). Thus equation (2) and (9) can be equated to obtain

$$E_6 \cos\left[\omega_B t + \phi_B + \phi_1(t) + \phi_2(t)\right] = E_6 \cos\left[\omega_B t + \phi_B + \Theta_o(t)\right], \tag{10}$$

which reduces to

$$\phi_1(t) + \phi_2(t) = \Theta_o(t) . \tag{11}$$

In Laplace transform notation, equation (11) becomes

$$\phi_1(S) + \phi_2(S) = \Theta_o(S) . \tag{12}$$

The change in phase of the VCO output is proportional to the integral of the input.[1,2] Thus, in Laplace transform notation, $\phi_1(S)$ becomes

$$\phi_1(S) = \frac{K_2'}{S} E_4(S) = \frac{K_2'}{S} \tau_{f2}(S) \frac{E_1 E_6}{2} \left[\theta_{in}(S) - \theta_o(S)\right] , \tag{13}$$

where K_2' is the constant of proportionality of the VCO. The phase added to e_5 in the phase modulator is directly proportional to e_3.[3] Thus, $\phi_2(S)$ becomes

$$\phi_2(S) = K_p E_3(S) = K_p K_1' \left[1 - \tau_{f1}(S)\right] \frac{E_1 E_6}{2} \left[\theta_{in}(S) - \theta_o(S)\right] , \tag{14}$$

where K_p is the constant of proportionality associated with the phase modulator.

Substitution of equations (13) and (14) into (12) yields

$$\begin{aligned} \theta_o(S) = {} & \frac{K_2'}{S} \tau_{f2}(S) \frac{E_1 E_6}{2} \left[\theta_{in}(S) - \theta_o(S)\right] + \\ & + K_p K_1' \left[1 - \tau_{f1}(S)\right] \frac{E_1 E_6}{2} \left[\theta_{in}(S) - \theta_o(S)\right] . \end{aligned} \tag{15}$$

If K_2 is defined as

$$K_2 = K_2' \frac{E_1 E_6}{2} , \tag{16}$$

and K_1 as

$$K_1 = K_p K_1' \frac{E_1 E_6}{2} , \tag{17}$$

equation (15) becomes

$$\theta_o(S) = K_2 \tau_{f2}(S) \left[\theta_{in}(S) - \theta_o(S)\right] + K_1\left[1 - \tau_{f1}(S)\right]\left[\theta_{in}(S) - \theta_o(S)\right]. \tag{18}$$

The solution of equation (18) for θ_o/θ_{in} yields

$$\frac{\theta_o(S)}{\theta_{in}(S)} = \frac{\frac{K_2}{S} \tau_{f2}(S) + K_1 \left[1 - \tau_{f1}(S)\right]}{1 + \frac{K_2}{S} \tau_{f2}(S) + K_1 \left[1 - \tau_{f1}(S)\right]} , \tag{19}$$

which is the general closed loop transfer function of the system.

The low pass filter in the VCO loop will be taken to be of the form of a standard tracking filter. That is, τ_{f2} is of the form

$$\tau_{f2}(S) = 1 + \frac{1}{S\tau_2} = \frac{S\tau_2 + 1}{S\tau_2} . \tag{20}$$

The low pass filter in the differential phase feedback loop will be taken to be of the form

$$\tau_{f1}(S) = \frac{1}{S\tau_1 + 1} \quad . \tag{21}$$

Substitution of equations (20) and (21) into the general closed loop transfer function yields

$$\frac{\theta_o(S)}{\theta_{in}(S)} = \frac{\frac{K_2}{S}\left[\frac{S\tau_2 + 1}{S\tau_2}\right] + K_1\left[\frac{S\tau_1}{S\tau_1 + 1}\right]}{1 + \frac{K_2}{S}\left[\frac{S\tau_2 + 1}{S\tau_2}\right] + K_1\left[\frac{S\tau_1}{S\tau_1 + 1}\right]} \quad . \tag{22}$$

This equation is in the form of

$$\frac{\theta_o(S)}{\theta_{in}(S)} = \frac{G(S)}{1 + G(S)} , \tag{23}$$

where

$$G(S) = \frac{K_2}{S}\left[\frac{S\tau_2 + 1}{S\tau_2}\right] + K_1\left[\frac{S\tau_1}{S\tau_1 + 1}\right] , \tag{24}$$

and may be represented by the block diagram shown in Figure 2. Thus a mathematical model has been obtained.

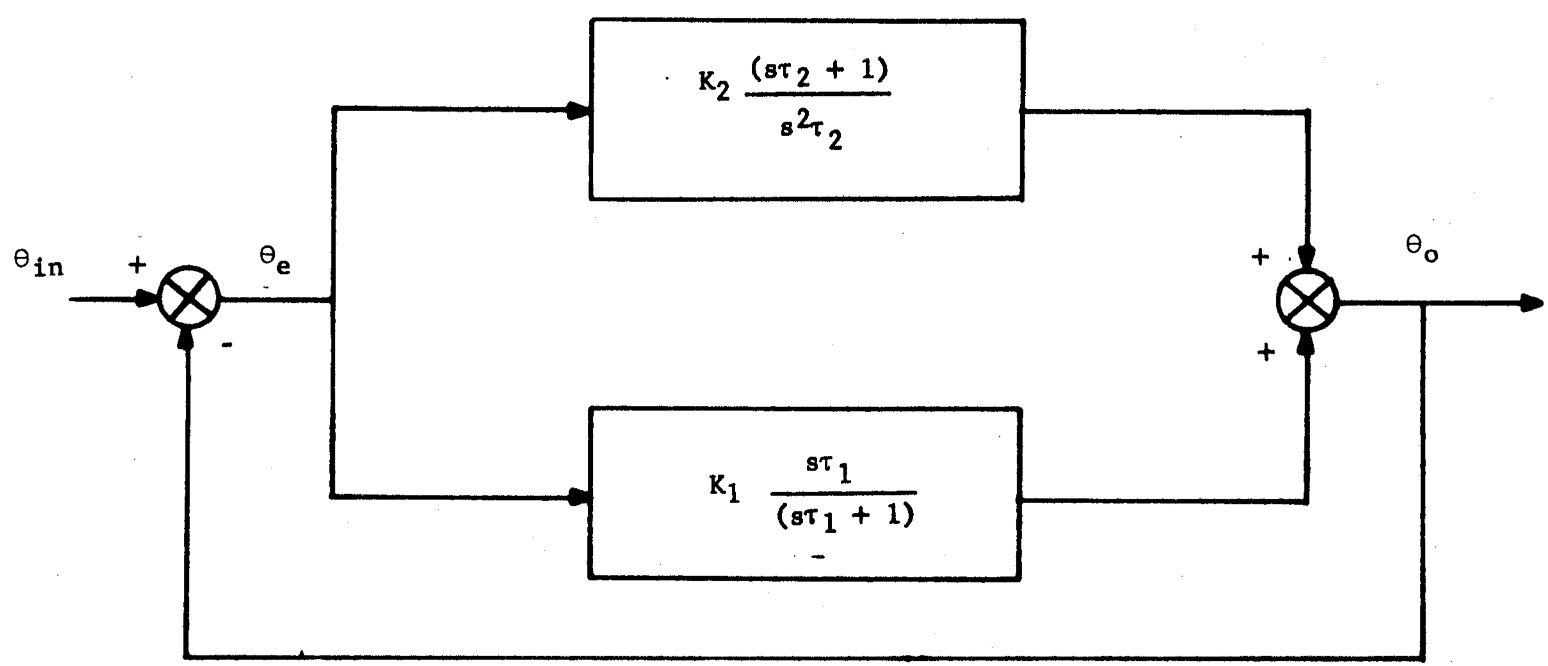


Fig. 2. Mathematical model of a phase locked receiver employing differential phase feedback.

III. SYSTEM ANALYSIS

A mathematical model of the system has been obtained and is given in Figure 2. This model which employs multiple loop feedback may be analyzed using linear feedback control theory. The root locus method, which is one of the methods available to analyze feedback control systems, indicates the effect of gain changes and the degree of stability of the system. In general, this information along with the initial and final values of the system output is sufficient for predicting the system performance.

The open loop transfer function of the system depicted in Figure 2 is

$$\frac{\theta_o(S)}{\theta_e(S)} = \frac{K_1\left[S^3 + \frac{K_2}{K_1} S^2 + \frac{K_2}{K_1}\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right) S + \frac{K_2}{K_1\tau_1\tau_2}\right]}{S^2(S + \frac{1}{\tau_1})} \quad . \tag{25}$$

A root locus diagram of the open loop transfer function of (25) can be obtained if the numerator of the equation can be reduced to a factored form.

Any polynomial of the form

$$P^N + a_{N-1}P^{N-1} + \ldots a_1P^1 + a_o = 0 \ , \tag{26}$$

can be factored with the aid of a root locus method.[4] In this case, the equation to be factored is

$$s^3 + \frac{K_2}{K_1} s^2 + \frac{K_2}{K_1}\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)s + \frac{K_2}{K_1\tau_1\tau_2} = 0 , \tag{27}$$

which is the numerator term of equation (25). Equation (27) can be rewritten as

$$1 + \frac{\frac{K_2}{K_1}\left[s^2 + \left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)s + \frac{1}{\tau_1\tau_2}\right]}{s^3} = 0 . \tag{28}$$

The numerator of the second term of equation (28) can be reduced to the factored form

$$1 + \frac{\frac{K_2}{K_1}\left(s + \frac{1}{\tau_2}\right)\left(s + \frac{1}{\tau_1}\right)}{s^3} = 0 , \tag{29}$$

which is in the standard form utilized in the root locus method of control system analysis. The root locus diagram of this equation is a plot of the roots of equation (27) as a function of K_2/K_1. The values chosen for τ_1 and τ_2 are relative quantities. Therefore, only the ratio of τ_1 to τ_2 is of importance since the values may be scaled to fit an actual system. For convenience, the radian bandwidth of the VCO loop, $\frac{1}{\tau_2}$, will be fixed at $\tau_2 = 1$ and root locus diagrams of equation (29) will be constructed for various values of the radian bandwidth of the differential phase feedback loop, $\frac{1}{\tau_1}$. Since the shape of the diagram is not affected by the value of τ_1, it is necessary to choose only three values to include all possible forms of the diagrams. The values chosen for τ_1 are 0.1, 1.0 and 10.0 and represent bandwidths of the differential phase feedback loop which are less than, equal to, and greater than the bandwidth of the VCO loop. These diagrams are given in Figure 3 and represent the roots of equation (27) as a function of K_2/K_1. From each diagram it can be seen that there exist three possible combinations for the roots: one real and two complex roots, all real with two equal roots, and all real and unequal roots. The form of the roots is determined by the value of K_2/K_1.

Now that the factors of the numerator of equation (25) have been determined, a root locus diagram of the open loop transfer function as a function of K_1 can be constructed. Since there are three possible combinations of the roots of equation (27) for each τ_1, three root locus diagrams of the open loop transfer are made for each value of τ_1. These diagrams are given in Figures 4 through 6.

If the system is to be stable for all values of gain, K_1, it is necessary to have all the zeroes and poles of equation (25) in the left half S-plane. Since this condition is always fulfilled by the poles, it is only necessary to be concerned with the value of the zeroes. From the root locus diagrams used to factor the numerator of

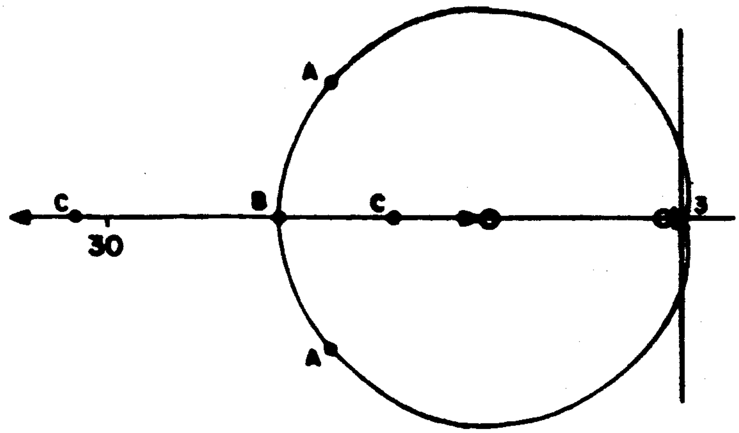
$\frac{\tau_1}{\tau_2} = 0.1$

(A)

A $\frac{K_2}{K_1} = 37.0$

B $\frac{K_2}{K_1} = 42.5$

C $\frac{K_2}{K_1} = 48.2$



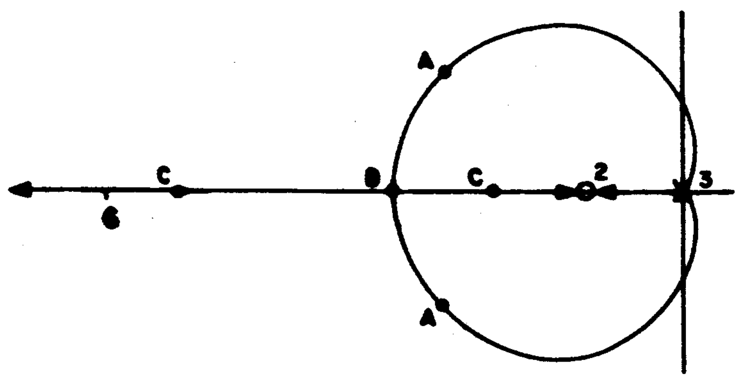
$\frac{\tau_1}{\tau_2} = 1.0$

(B)

A $\frac{K_2}{K_1} = 5.75$

B $\frac{K_2}{K_1} = 6.75$

C $\frac{K_2}{K_1} = 8.0$



$\frac{\tau_1}{\tau_2} = 10.0$

(C)

A $\frac{K_2}{K_1} = 3.70$

B $\frac{K_2}{K_1} = 4.28$

C $\frac{K_2}{K_1} = 4.82$

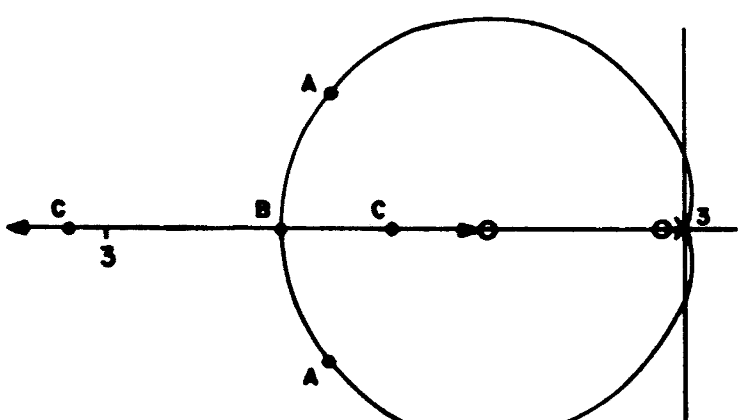


Fig. 4. Root locus diagram of equation (29).

(A) $\frac{K_2}{K_1} = 37.0$

(B) $\frac{K_2}{K_1} = 42.5$

(C) $\frac{K_2}{K_1} = 48.2$

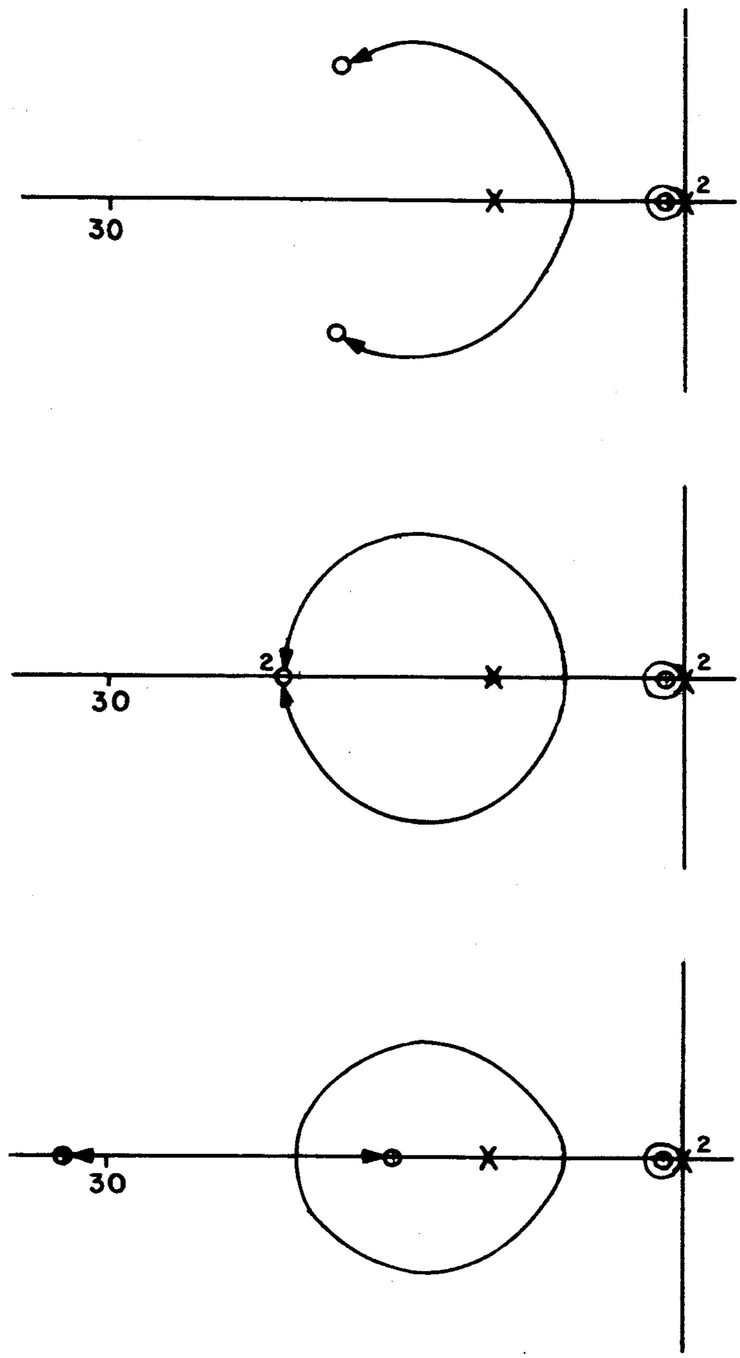


Fig. 4. Root locus diagram of equation (25) for $\frac{\tau_1}{\tau_2} = 0.1$

(A) $\frac{K_2}{K_1} = 5.75$

(B) $\frac{K_2}{K_1} = 6.75$

(C) $\frac{K_2}{K_1} = 8.0$

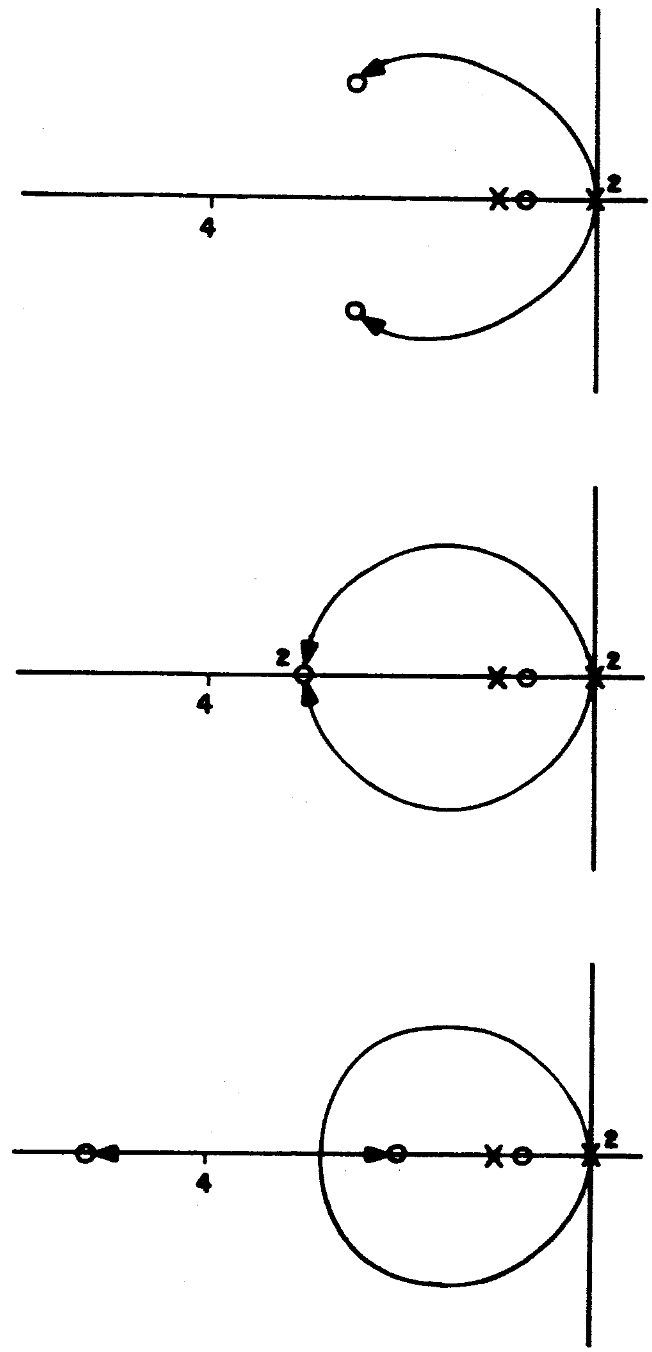


Fig. 5. Root locus diagram of equation (25) for $\frac{\tau_1}{\tau_2} = 1.0$.

(A)

$\frac{K_2}{K_1} = 3.70$

(B)

$\frac{K_2}{K_1} = 4.28$

(C)

$\frac{K_2}{K_1} = 4.82$

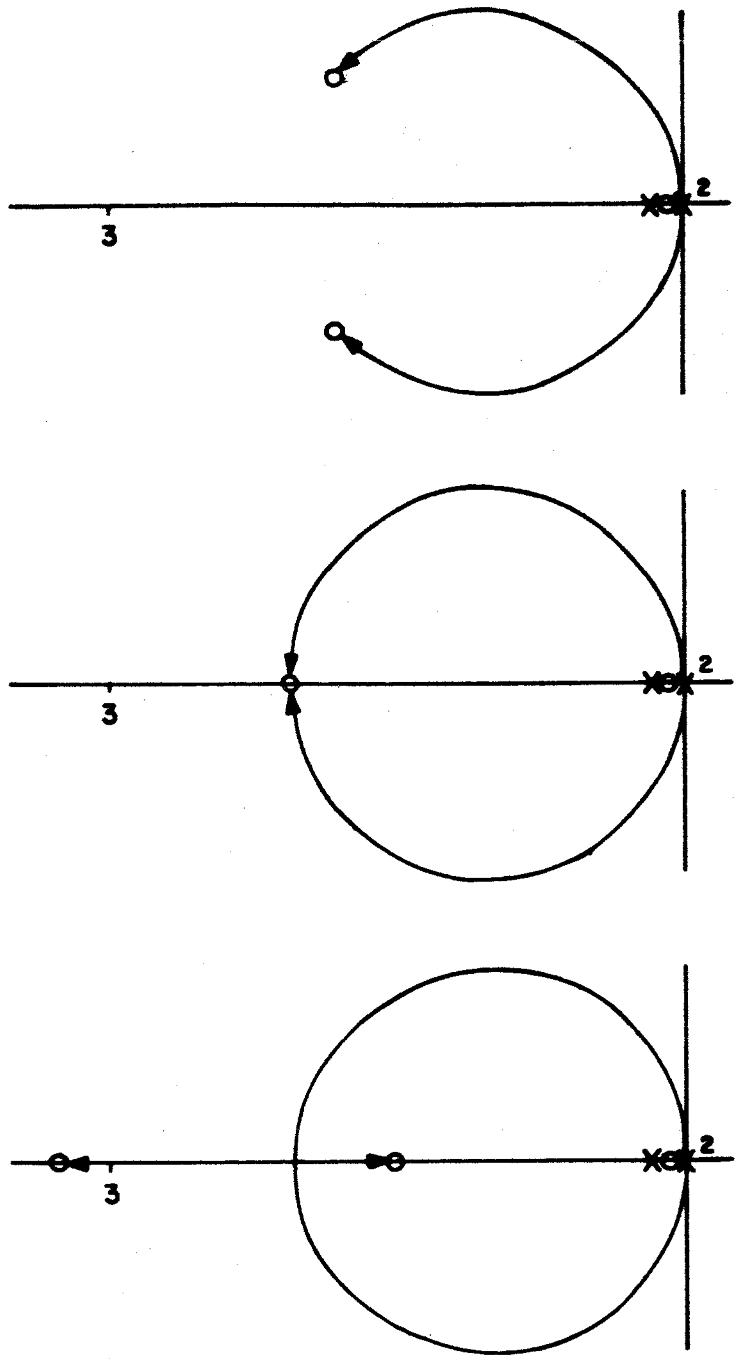


Fig. 6. Root locus diagram of equation (25) for $\frac{\tau_1}{\tau_2} = 10.0$.

equation (25) it can be seen that there exists a minimum value of the ratio of gains K_2/K_1 to insure that all zeroes have negative real parts. This required ratio is a function of τ_1/τ_2 and decreases as the ratio of τ_1/τ_2 increases. Thus for ratios of K_2/K_1 greater than the required minimum, the system is unconditionally stable.

From the root locus diagrams of open loop transfer function, it can be seen that ratios of τ_1/τ_2 less than one have a greater effect upon the response time of the system than do ratios greater than one. The root locus diagram for τ_1 = 1.0 and τ_1 = 10.0 are essestially the same, while the diagram for τ_1 = 0.1 is moved away from the $j\omega$ axis thus decreasing the damping times. Thus for a fast response time of the VCO, a small ratio of τ_1/τ_2 is needed.

The initial value of the time response of any system is defined by

$$\lim_{t \to 0} f(t) = \lim_{S \to \infty} SF(S). \tag{30}$$

Hence, for a step input the initial value of $\theta_o(t)$ is given by

$$\lim_{t \to 0} \theta_o(t) = \lim_{S \to \infty} \frac{\theta_{in}K_1\left[S^3 + \frac{K_2}{K_1}S^2 + \frac{K_2}{K_1}\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)S + \frac{K_2}{K_1\tau_1\tau_2}\right]}{S^2\left(S+\frac{1}{\tau_1}\right)+K_1\left[S^3+\frac{K_2}{K_1}S^2+\frac{K_2}{K_1}\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)S + \frac{K_2}{K_1\tau_1\tau_2}\right]}, \tag{31}$$

where θ_{in} is the magnitude of the step input. Equation (31) reduces to

$$\theta_o(0^+) = \frac{K_1}{1 + K_1}\,\theta_{in} \;. \tag{32}$$

Therefore, K_1 must be large for the initial value of $\theta_o(t)$ to be within a small percentage of θ_{in}. For $\theta_o(t)$ to be within 5% of θ_{in}, K_1 must be greater than or equal to 19.

The final value of the time response is defined by

$$\lim_{t \to \infty} f(t) = \lim_{S \to 0} SF(S) \;. \tag{33}$$

Hence, the final value of $\theta_o(t)$ is given by

$$\lim_{t \to \infty} \theta_o(t) = \lim_{S \to 0} \frac{\theta_{in}K_1\left[S^3 + \frac{K_2}{K_1}S^2 + \frac{K_2}{K_1}\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)S + \frac{K_1}{K_2\tau_1\tau_2}\right]}{S^2\left(S+\frac{1}{\tau_1}\right) + K_1\left[S^3 + \frac{K_2}{K_1}S^2 + \frac{K_2}{K_1}\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)S + \frac{K_1}{K_2\tau_1\tau_2}\right]} \;, \tag{34}$$

which reduces to

$$\theta_o(\infty) = \theta_{in}. \tag{35}$$

Since the output of the VCO is of importance in determining the stability of the VCO, it is necessary to calculate the initial and final values of the VCO output in addition to those of the system output.

The initial and final values of the VCO output, θ_{vco}, can be calculated if a function relating θ_{vco} to the system input can be determined. This function can be obtained from Figure 7, where this figure is obtained from Figure 2 by block diagram manipulation. Hence, the closed loop transfer function of the VCO is

$$\frac{\theta_{vco}(S)}{\theta_{in}(S)} = \frac{K_2 S^2 + K_2\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right) S + \frac{K_2}{\tau_1\tau_2}}{(K_1 + 1)S^3 + (K_2 + \frac{1}{\tau_1})S^2 + K_2\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)S + \frac{K_2}{\tau_1\tau_2}} . \tag{36}$$

For a step input, equation (36) becomes

$$\theta_{vco}(S) = \frac{\theta_{in}\left[K_2 S^2 + K_2\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right) S + \frac{K_2}{\tau_1\tau_2}\right]}{S\left[(K_1 + 1)S^3 + (K_2 + \frac{1}{\tau_1})S^2 + K_2\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)S + \frac{K_2}{\tau_1\tau_2}\right]} . \tag{37}$$

Substitution of equation (37) into equation (30) yields the initial values of $\theta_{vco}(t)$ as

$$\theta_{vco}(0^+) = 0. \tag{38}$$

Substitution of equation (37) into equation (34) yields the final value of $\theta_{vco}(t)$ as

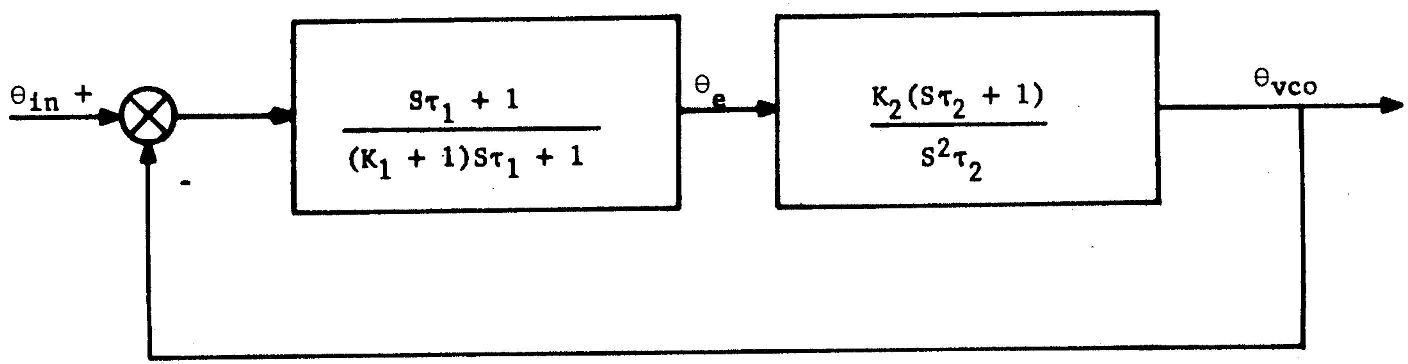


Fig. 7. Mathematical model of a phase locked receiver employing differential phase feedback with the output of the VCO as the system output.

$$\theta_{vco}(\infty) = \theta_{in}. \tag{39}$$

The information contained in the root locus diagrams and the initial and final values of the system response indicates that the desired system performance can be obtained. That is, the composite output, θ_o, tracks the input within a percentage determined by the value of K_1 while the output of the VCO, θ_{vco} is zero initially and reaches the value of the input only if the input remains constant for a period of time determined by the loop parameters.

The complete time response for θ_o and θ_{vco} can be calculated by obtaining functions for θ_o and θ_{vco} in Laplace transformation notation and determining the inverse Laplace transform.

For a one radian step input, $\theta_o(S)$ is given by

$$\theta_o(S) = \left[\frac{1}{S}\right] \frac{K_1S^3 + K_2S^2 + K_2\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)S + \frac{K_2}{\tau_1\tau_2}}{(K_1 + 1)S^3 + (K_2 + \frac{1}{\tau_1})S^2 + K_2\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)S + \frac{K_2}{\tau_1\tau_2}}. \tag{40}$$

The denominator of this equation can be arranged in factored form and the numerator rearranged to obtain

$$\theta_o(S) = \frac{1 + AS + BS^2 + CS^3}{S(1 + T_1S)\,(1 + T_2S)\,(1 + T_3S)}\ , \tag{41}$$

where $A = \tau_1\tau_2 \left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)$,

$B = \tau_1\tau_2$,

$C = \tau_1\tau_2 K_1/K_2$,

$T_1 = 1/R_1$

$T_2 = 1/R_2$

$T_3 = 1/R_3$

and R_1, R_2, and R_3 are the roots of the denominator of equation (40). The inverse Laplace transform of equation (41) is

$$\theta_o(t) = 1 - \frac{T_1^{\,3} - AT_1^{\,2} + BT_1 - C}{T_1(T_1 - T_2)(T_1 - T_3)} e^{-t/T_1} - \frac{T_2^{\,3} - AT_2^{\,2} + BT_2 - C}{T_2(T_2 - T_1)(T_2 - T_3)} e^{-t/T_2} - \frac{T_3^{\,3} - AT_3^{\,2} + BT_3 - C}{T_3(T_3 - T_1)(T_3 - T_2)} e^{-t/T_3} . \tag{42}$$

$\theta_o(t)$ was calculated with the aid of a digital computer for the system as represented by each root locus diagram. The computer program is given in Appendix A. The values of RT_1, RT_2, and RT_3 were determined from each diagram for $K_1 = 20$. Figure 8 is a typical response for $\theta_o(t)$, with the complete results given in Appendix B.

The output of the VCO, θ_{vco}, for a one radian step input is

$$\theta_{vco}(S) = \left[\frac{1}{S} \right] \frac{K_2 S^2 + K_2\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)S + \frac{K_2}{\tau_1\tau_2}}{(K_1 + 1)S^3 + (K_2 + \frac{1}{\tau_1})S^3 + K_2(\frac{1}{\tau_1} + \frac{1}{\tau_2})S + \frac{K_2}{\tau_1\tau_2}} , \tag{43}$$

which can be rewritten as

$$\theta_{vco}(S) = \frac{1 + AS + BS^2}{S(1 + T_1S)(1 + T_2S)(1 + T_2S)} . \tag{44}$$

The inverse Laplace transform of equation (44) is

$$\theta_{vco}(t) = 1 - \frac{T_1{}^2 - AT_1 + B}{(T_1 - T_2)(T_1 - T_3)} e^{-t/T_1}$$

$$- \frac{T_2{}^2 - AT_2 + B}{(T_2 - T_1)(T_2 - T_3)} e^{-t/T_2} - \frac{T_3^2 - AT_3 + B}{(T_3 - T_1)(T_3 - T_2)} e^{-t/T_3} . \tag{45}$$

This equation was programmed on a digital computer to give θ_{vco} as a function of time for the same cases as θ_o. The response for θ_{vco} for each case is given in the same figure with the corresponding response of θ_o.

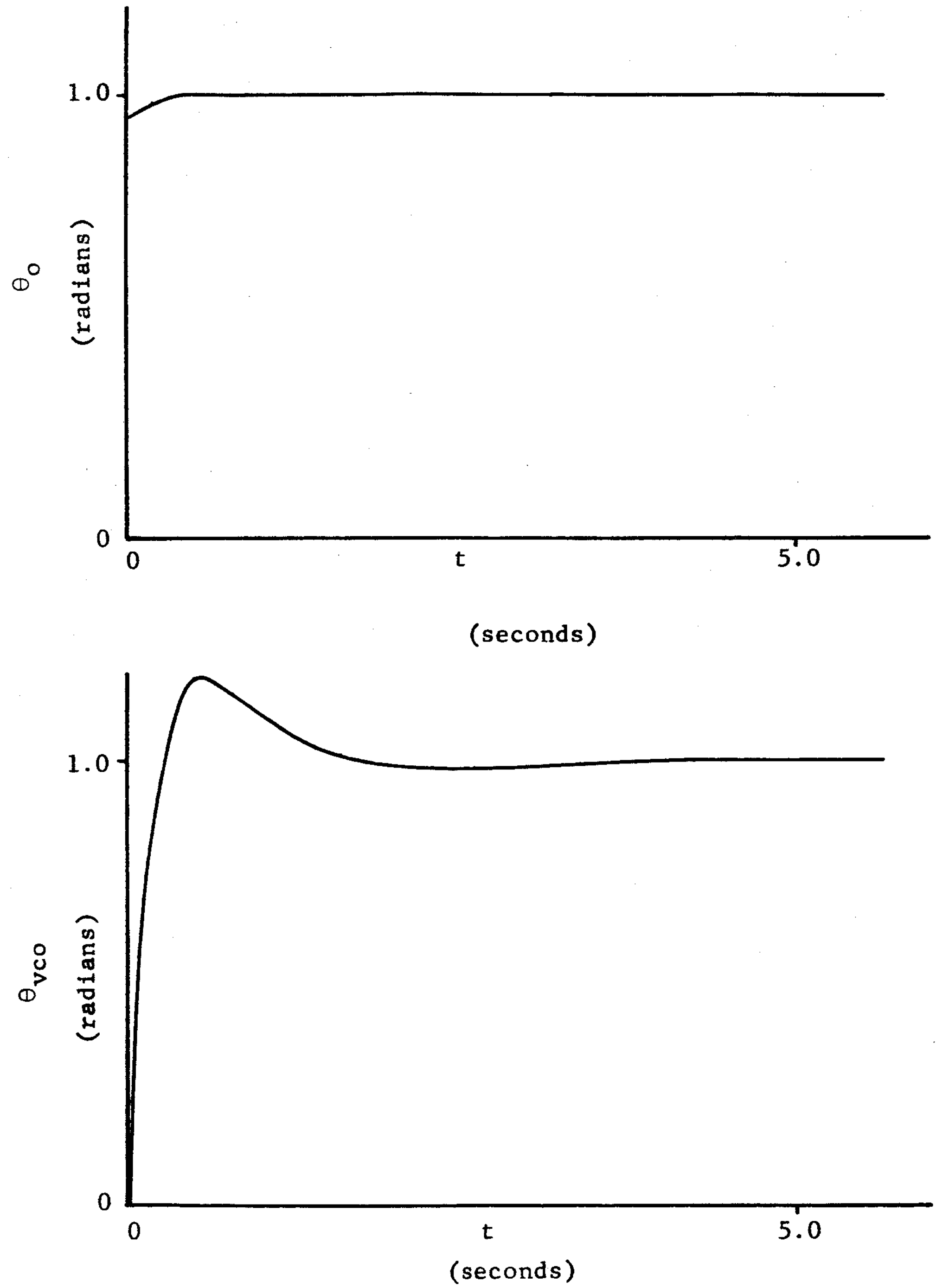


Fig. 8. Typical response of Θ_o and Θ_{vco} with a one radian step input.

From the output response curves given in Figure B-1 through B-9, it can be seen that the response time decreases as the ratio of τ_1/τ_2 decreases as was expected. As the value of K_2/K_1 increases, the value of the overshoot in the VCO output decreases while the response time is not changed by an appreciable amount. As predicted, the response of the system output is within 5% of the input at all times. Thus the desired system performance has been obtained.

IV. CONCLUSION

It has been shown that the addition of differential phase feedback to a standard phase-lock receiver results in a system which has an output that tracks the input signal dynamics while maintaining a stable VCO. The ability of the system to track the input signal is dependent upon the value of K_1. Equation (32) indicates that K_1 should be large to minimize the initial error caused by a change in the input signal. The stability of the VCO is dependent upon the ratio K_2/K_1 and τ_1/τ_2.

If the value of K_2/K_1 is greater than the minimum required to insure unconditional stability, then the ratio, τ_1/τ_2, is the predominate control on the response time of the VCO. Figures B-1 through B-9 indicate that the ratio, K_2/K_1, has little effect upon the response time.

Since the ability of the system to track and the stability of the VCO are dependent upon different system parameters, the two may be adjusted independently. It can be shown that this is not true in the case of the standard phase-lock receiver. The two system characteristics are dependent upon the same parameters. A standard receiver which has the desired tracking capability will not have a stable VCO while the system with a stable VCO will not track rapidly changing input signals. Therefore to obtain the desired characteristics, another type system, such as the system employing differential phase feedback, must be used.

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2. A. V. Balaknishnan (ed.), "Space Communications," McGraw Hill Book Company, Inc., New York, N. Y.; 1963.

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APPENDIX A

DIGITAL COMPUTER PROGRAM USED TO CALCULATE OUTPUT RESPONSE OF THE SYSTEM

TABLE 1

DEFINITIONS OF COMPUTER PROGRAM SYMBOLS

PROGRAM SYMBOL	VARIABLE
T1	τ_1
T2	τ_2
G1	K_1
G2	K_2
TC1	T_1
TC2	T_2
TC3	T_3
RT1	R_1
RT2	R_2
RT3	R_3
THETAO	θ_o
THETAV	θ_{vco}
T(I)	t(time)
ALPHA	Real part of R_1 and R_2
BETA	Imaginary part of R_1 and R_2

```
C      IBM 7040 FORTRAN IV DIGITAL COMPUTER PROGRAM
C      TO CALCULATE OUTPUT OF A PHASE LOCK RECEIVER
C      EMPLOYING DIFFERENTIAL PHASE FEEDBACK


      DIMENSION  T(50), X(50), Y(50), Z(50)
      TIME = 0.
      DO 5 I=1,50
      T(I) = TIME
    5 TIME = TIME + 0.02
      G1 = 20.
      T2 = 1.0
      DO 61 L=1,3
      DO 21 M=1,2
      READ (5,100) G2, T1, ALPHA, BETA, RT3
  100 FORMAT (5F10.0)
      WRITE (6,150) G1, T1, G2, T2, ALPHA, BETA, RT3
  150 FORMAT(////1H , 3HG1=,F4.1,4X,3HT1=,F4.1,4X,3HG2=,
     1  F5.1,4X,3HT2=,F4.1,4X,6HALPHA=,F7.4,4X,5HBETA=,
     2  F7.4,4X,4HRT3=,F6.4 / )
      RE = ALPHA / (ALPHA**2 + BETA**2 )
      XI = - BETA * RE / ALPHA
      TC3 = 1. / RT3
      A = T1 * T2 * ( 1. / T1 + 1. / T2 )
      B = T1 * T2
      C = G1 * T1* T2 / G2
      D1  = RE**3-A*RE**2+(A-3.*RE)*XI**2+B*RE-C
      D2  = 3.*RE**2*XI-XI**3-2.*A*RE*XI+B*XI
      D3  = XI**2*(2.*TC3 - 4.*RE)
      D4  = 2.*RE**2*XI-2.*XI**3-2.*RE*XI*TC3
      D5  = -RE/(RE**2+XI**2)
      D6  = XI*D5/RE
      D7  = D1*D3+D2*D4
      D8  = D2*D3-D1*D4
      D9  = 2./(D3**2+D4**2)
      D10 = (TC3**3-A*TC3**2+B*TC3-C)/(TC3*(TC3-RE)**2+XI
     1   **2*TC3)
      WRITE (6,175) D5, D6, D7, D9, D10
  175 FORMAT (  1X,4HD5 =,E16.7,5X,4HD6 =,E16.7,5X,4HD7 =
     1 ,E16.7,5X,4HD9 =,E16.7,5X,5HD10 =,E16.7 /)
      DO 15 I=1,31
      X(I) = -D9*EXP(D5*T(I))*(D7*COS(D6*T(I))+D8*SIN(D6*
     1 T(I)))
      IF(ABS(X(I)).LT..0001) GO TO 17
   15 CONTINUE
   17 CALL ZERO (X(I),X(31))
      DO 16 J=1,31
      Y(J) = -D10*EXP(-RT3*T(J))
      IF(ABS(Y(J)).LT..0001) GO TO 18
   16 CONTINUE
```

```
C       PROGRAM CONTINUED

   18 CALL ZERO (Y(J),Y(31))
      DO 10 K=1,31
      THETAO = 1.0 + X(K) + Y(K)
   10 WRITE (6,200) T(K), THETAO
  200 FORMAT(1H ,15X,4H T =,F4.1,20X,8HTHETAO =,F7.4)
      E1 = RE**2-XI**2-A*RE+B
      E2 = 2.*RE*XI-A*XI
      E4 = 2.*RE*XI-2.*XI*TC3
      E3 = -2.*XI**2
      E5 = D5
      E6 = D6
      E7 = E1*E3+E2*E4
      E8 = E2*E3-E1*E4
      E9 = 2./(E3**2+E4**2)
      E10 = (TC3**2-A*TC3+B)/((TC3-RE)**2+XI**2)
      WRITE (6,185) E5, E6, E7, E9, E10
  185 FORMAT (//1X,4HE5 =,E16.7,5X,4HE6 =,E16.7,5X,4HE7 =
     1 ,E16.7,5X,4HE9 =,E16.7,5X,5HE10 =,E16.7 /)
      DO515 I=1,31
      X(I) = -E9*EXP(E5*T(I))*(E7*COS(E6*T(I))+E8*SIN(E6*
     1 T(I)))
      IF(ABS(X(I)).LT..0001) GO TO517
  515 CONTINUE
  517 CALL ZERO (X(I),X(31))
      DO516 J=1,31
      Y(J) = -E10*EXP(-RT3*T(J))
      IF(ABS(Y(J)).LT..0001) GO TO   518
  516 CONTINUE
  518 CALL ZERO (Y(J),Y(31))
      DO510 K=1,31
      THETAV = 1.0 + X(K) + Y(K)
  510 WRITE (6,300) T(K), THETAV
   21 CONTINUE
  300 FORMAT (1H ,15X,4H T =,F4.1,20X,8HTHETAV =,F7.4)
      READ (5,100)  G2, T1, RT1, RT2, RT3
      WRITE (6,250) G1, T1, G2, T2, RT1, RT2, RT3
  250 FORMAT(////1H , 3HG1=,F4.1,4X,3HT1=,F4.1,4X,3HG2=,
     1 F5.1,4X,3HT2=,F4.1,4X,4HRT1=,F7.4,4X,4HRT2=,F7.4,
      2 4X,4HRT3=,F7.4)
      C = G1 * T1* T2 / G2
      TC1 = 1./RT1
      TC2 = 1./RT2
      TC3 = 1. / RT3
      H4 = (TC1**3-A*TC1**2+B*TC1-C)/(TC1*(TC1-TC2)*
     1  (TC1-TC3))
      H5 = (TC2**3-A*TC2**2+B*TC2-C)/(TC2*(TC2-TC1)*
     1  (TC2-TC3))
```

```
C     PROGRAM CONTINUED

      H6 = (TC3**3-A*TC3**2+B*TC3-C)/(TC3*(TC3-TC1)*
     1  (TC3-TC2))
      WRITE (6,275) H4, H5, H6
  275 FORMAT (/1X,4HH4 =,E16.7,5X,4HH5 =,E16.7,5X,4HH6 =,
     1  E16.7 /)
      DO 25 I=1,31
      X(I) = -H4*EXP(-RT1*T(I))
      IF(ABS(X(I)).LT..0001) GO TO 22
   25 CONTINUE
   22 CALL ZERO (X(I),X(31))
      DO 26 J=1,31
      Y(J) = -H5*EXP(-RT2*T(J))
      IF(ABS(Y(J)).LT..0001) GO TO 23
   26 CONTINUE
   23 CALL ZERO (Y(J),Y(31))
      DO 27 K=1,31
      Z(K) = -H6*EXP(-RT3*T(K))
      IF(ABS(Z(K)).LT..0001) GO TO  24
   27 CONTINUE
   24 CALL ZERO (Z(K),Z(31))
      DO 20 N=1,31
      THETAO = 1.0 + X(N) + Y(N) + Z(N)
   20 WRITE (6,200) T(N) , THETAO
      H7 = (TC1**2-A*TC1+B)/((TC1-TC2)*(TC1-TC3))
      H8 = (TC2**2-A*TC2+B)/((TC2-TC1)*(TC2-TC3))
      H9 = (TC3**2-A*TC3+B)/((TC3-TC1)*(TC3-TC2))
      WRITE (6,285) H7, H8, H9
  285 FORMAT (//1X,4HH7 =,E16.7,5X,4HH8 =,E16.7,5X,4HH9 =
     1  ,E16.7 / )
      DO525 I=1,31
      X(I) = -H7*EXP(-RT1*T(I))
      IF(ABS(X(I)).LT..0001) GO TO522
  525 CONTINUE
  522 CALL ZERO (X(I),X(31))
      DO526 J=1,31
      Y(J) = -H8*EXP(-RT2*T(J))
      IF(ABS(Y(J)).LT..0001) GO TO523
  526 CONTINUE
  523 CALL ZERO (Y(J),Y(31))
      DO527 K=1,31
      Z(K) = -H9*EXP(-RT3*T(K))
      IF(ABS(Z(K)).LT..0001) GO TO 524
  527 CONTINUE
  524 CALL ZERO (Z(K),Z(31))
      DO520 N=1,31
      THETAV = 1.0 + X(N) + Y(N) + Z(N)
  520 WRITE (6,300) T(N) , THETAV
```

```
C     PROGRAM CONTINUED

   61 CONTINUE
      STOP
      END
      SUBROUTINE ZERO (A(N),A(K))
      DIMENSION A(K)
      DO 10 I=N,K
   10 A(I) = 0.
      RETURN
      END
```

APPENDIX B

OUTPUT RESPONSE OF Θ_o AND Θ_{vco} FOR A ONE RADIAN STEP INPUT WITH $\tau_2 = 1.0$ and $K_1 = 20.0$

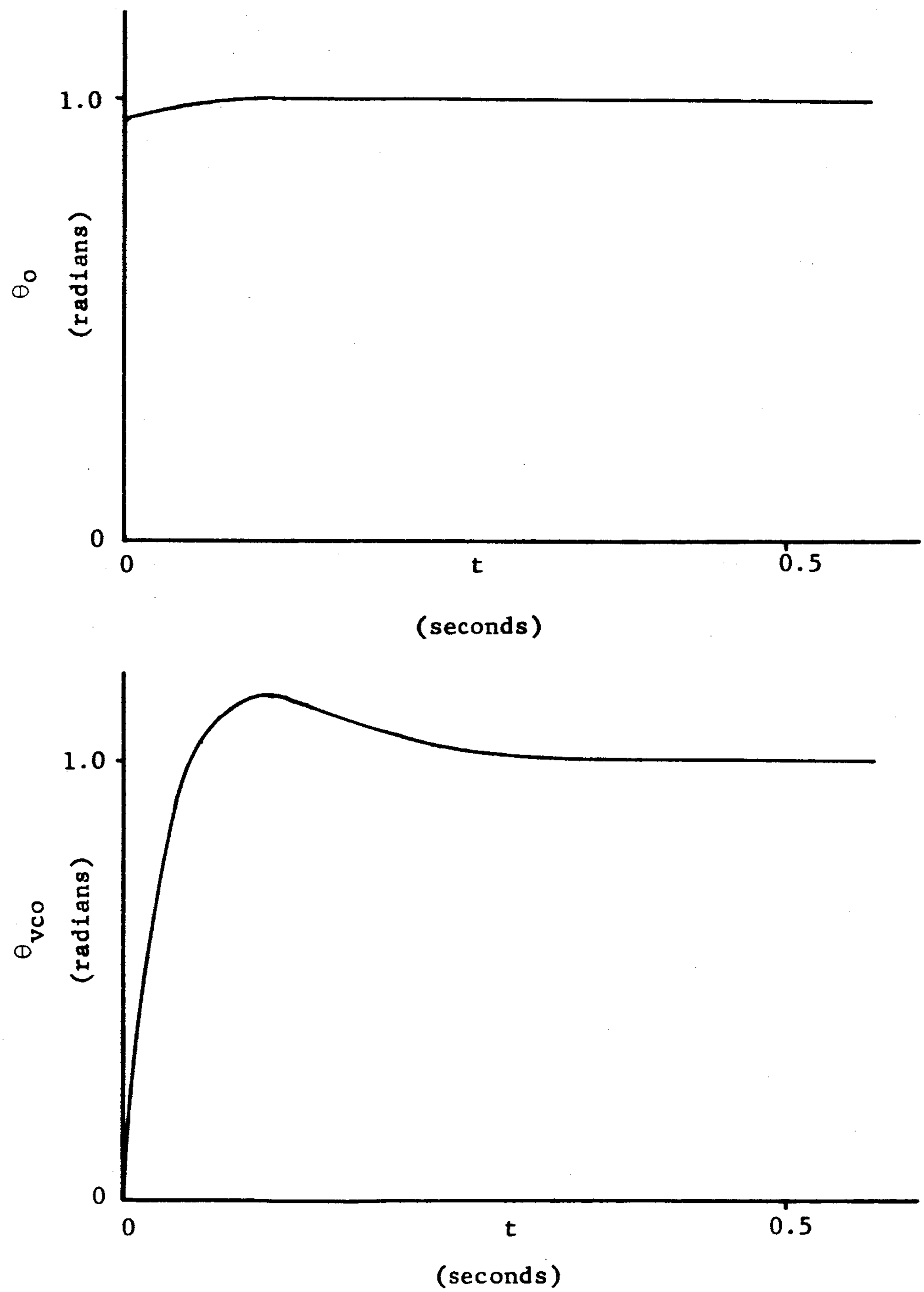


Fig. B-1. Response of θ_o and θ_{vco} with τ_1/τ_2=0.1 and K_2/K_1=37.0.

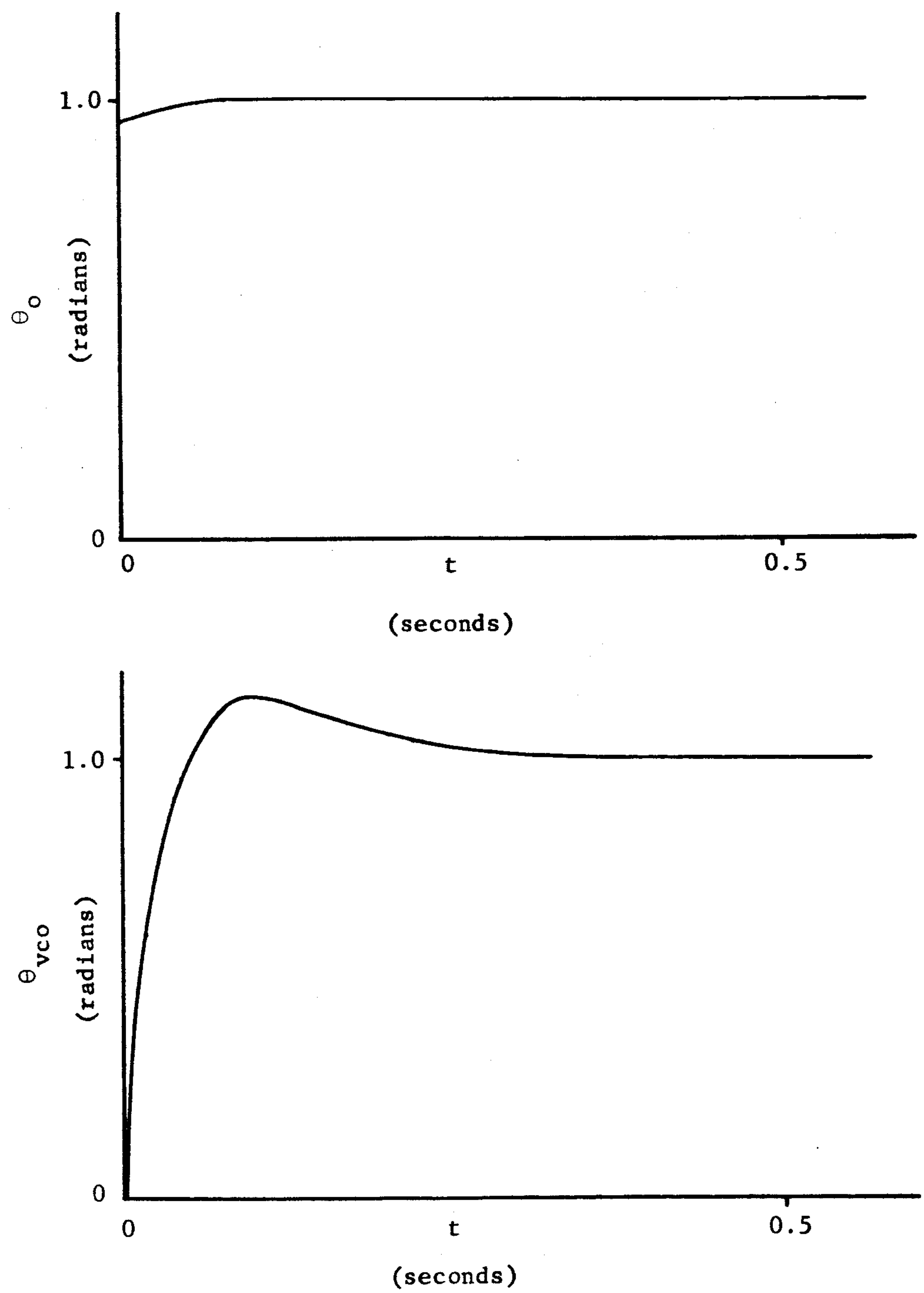


Fig. B-2. Response of θ_o and θ_{vco} with τ_1/τ_2=0.1 and K_2/K_1=42.5.

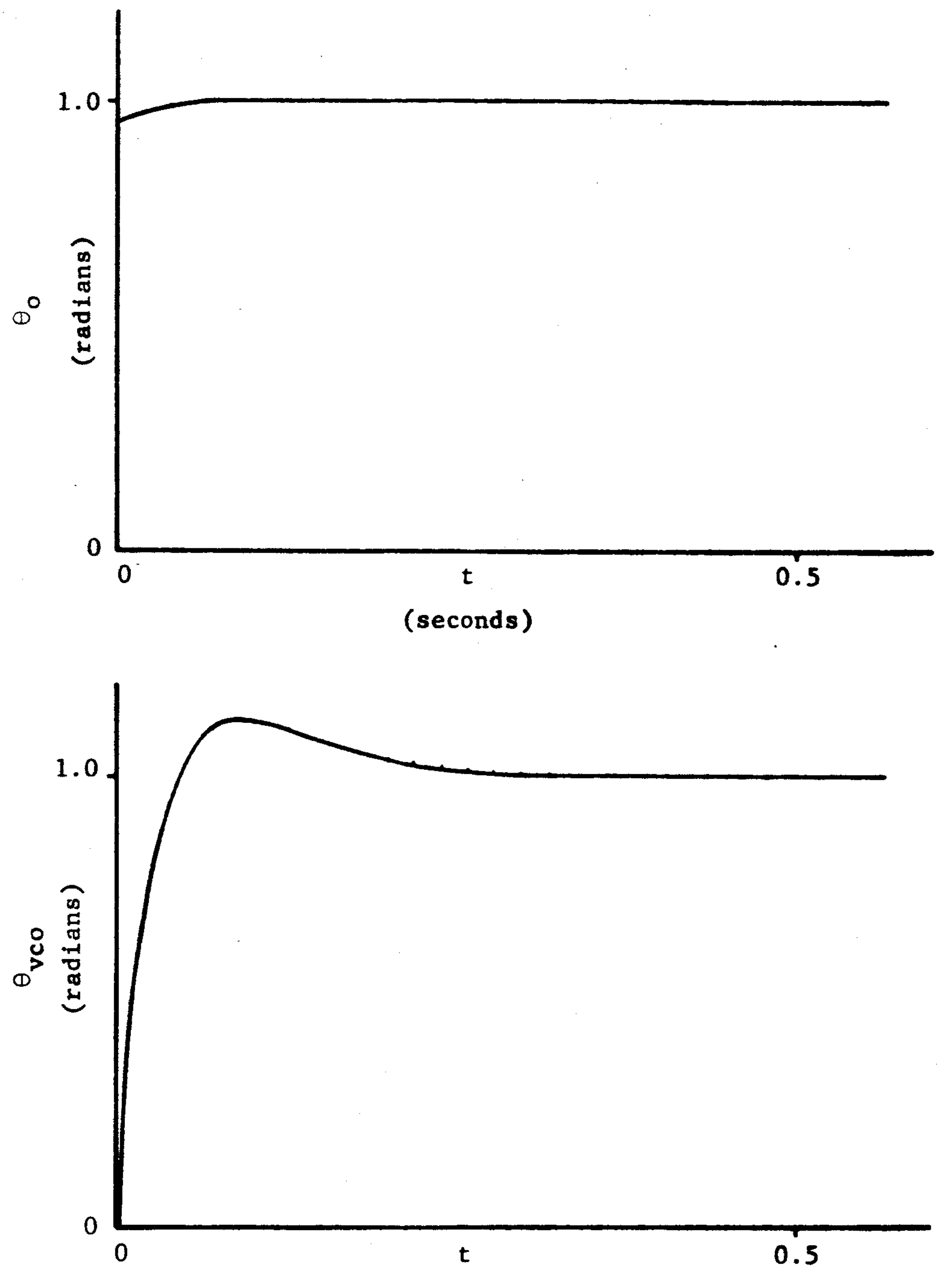
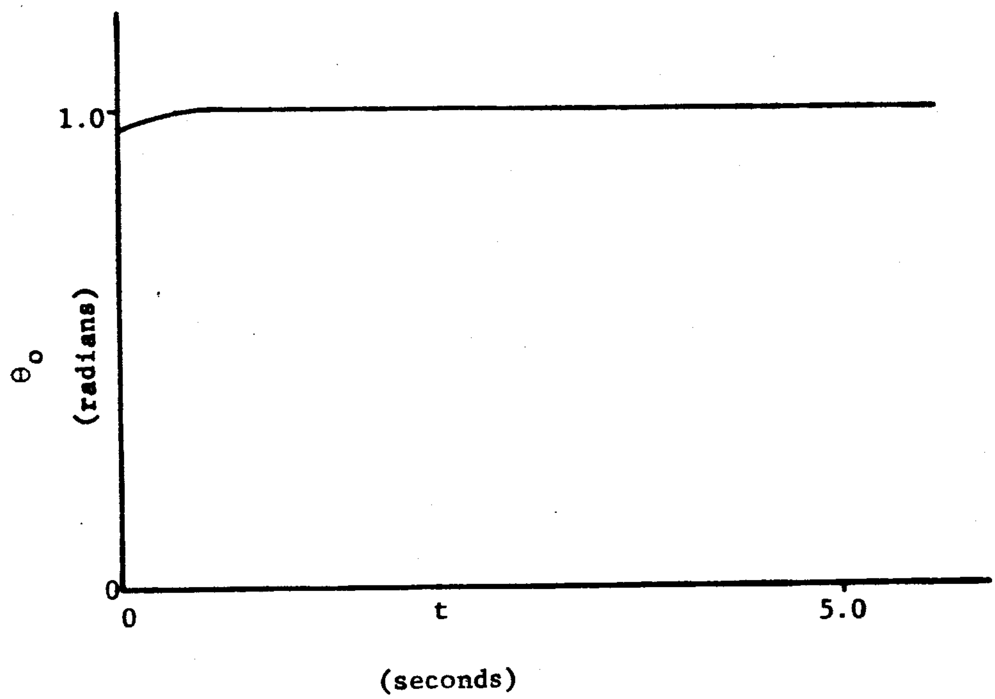


Fig. B-3. Response of θ_o and θ_{vco} with $\tau_1/\tau_2=0.1$ and $K_2/K_1=48.2$.



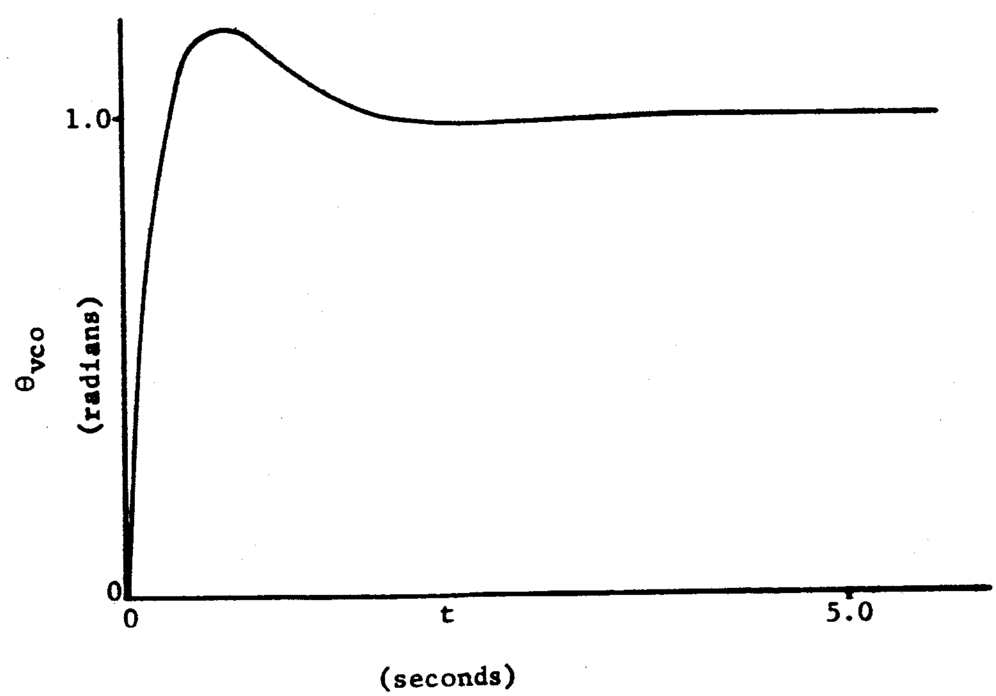


Fig. B-4. Response of θ_o and θ_{vco} with $\tau_1/\tau_2=1.0$ and $K_2/K_1=5.75$.

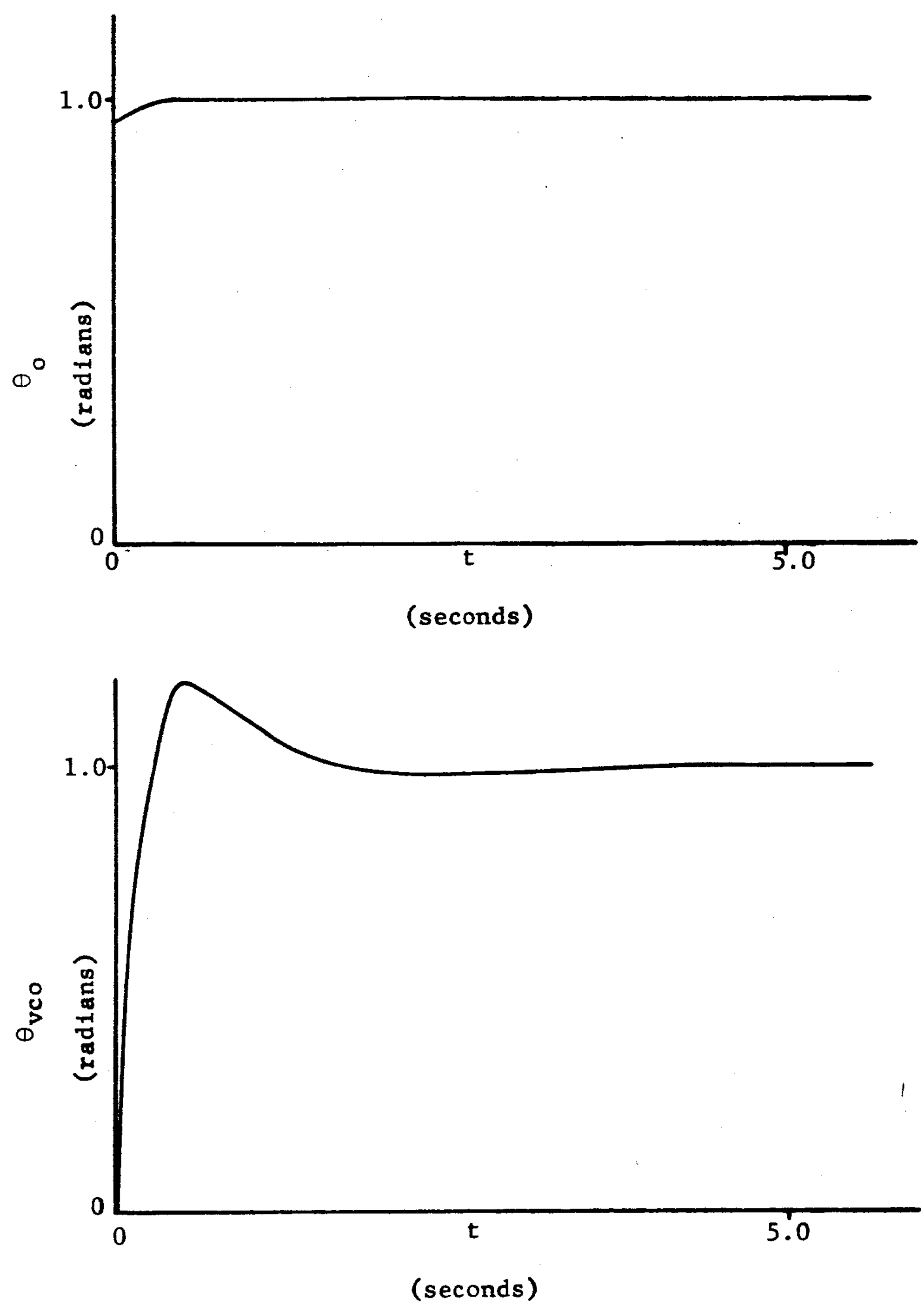


Fig. B-5. Response of Θ_o and Θ_{vco} with τ_1/τ_2=1.0 and K_2/K_1=6.75.

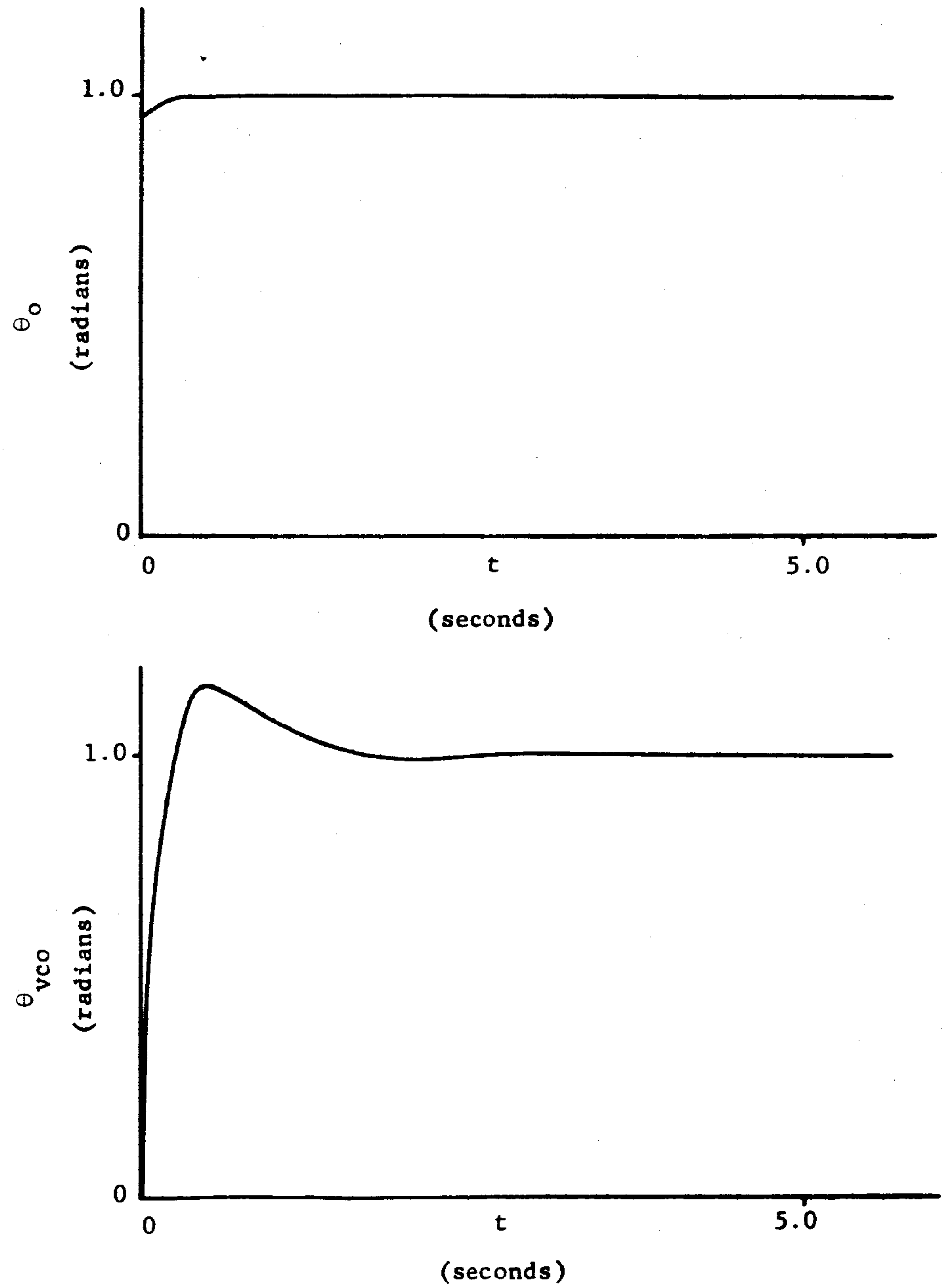


Fig. B-6. Response of Θ_o and Θ_{vco} with $\tau_1/\tau_2=1.0$ and $K_2/K_1=8$.

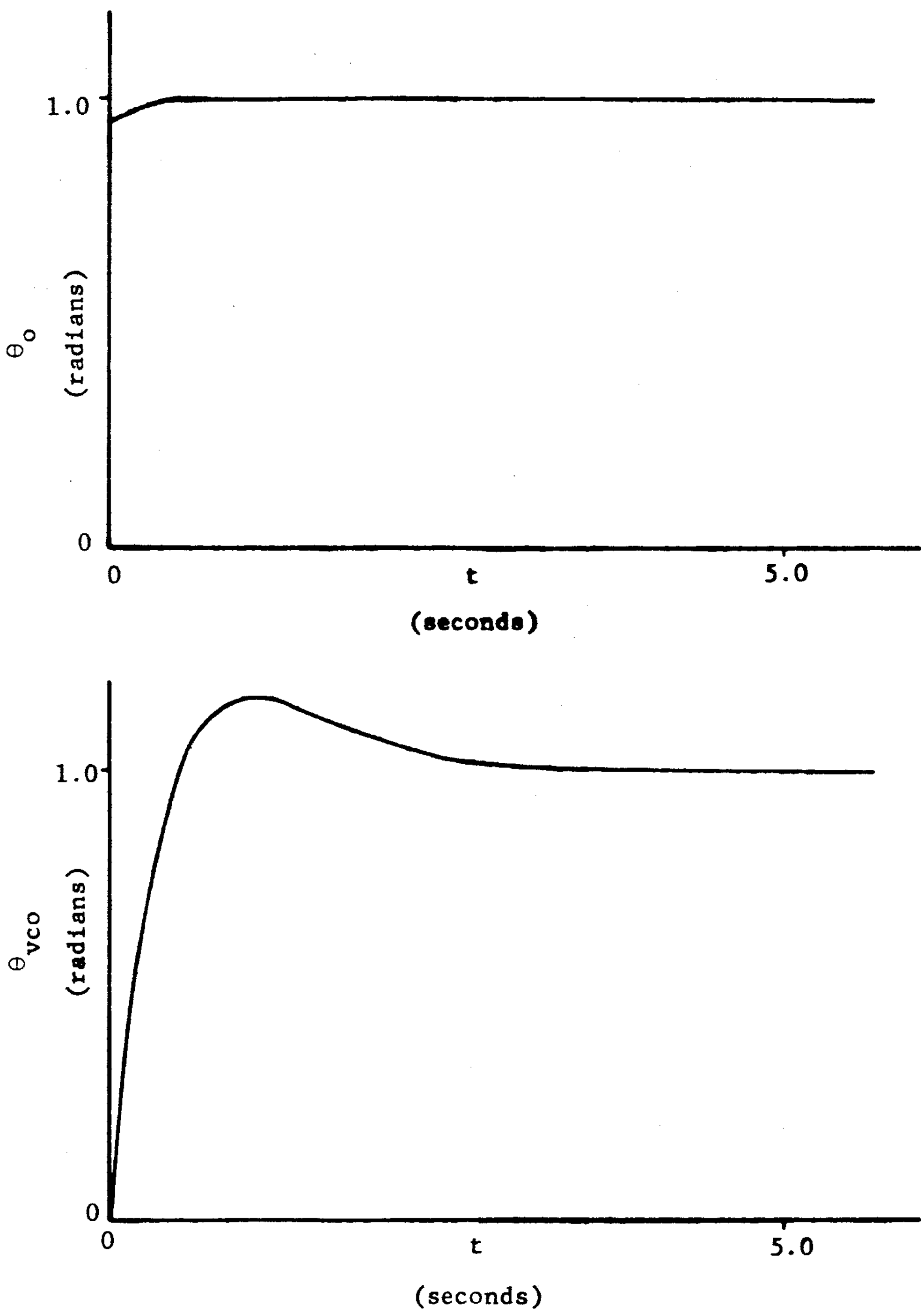


Fig. B-7. Response of Θ_o and Θ_{vco} with τ_1/τ_2=10.0 and K_2/K_1=3.7.

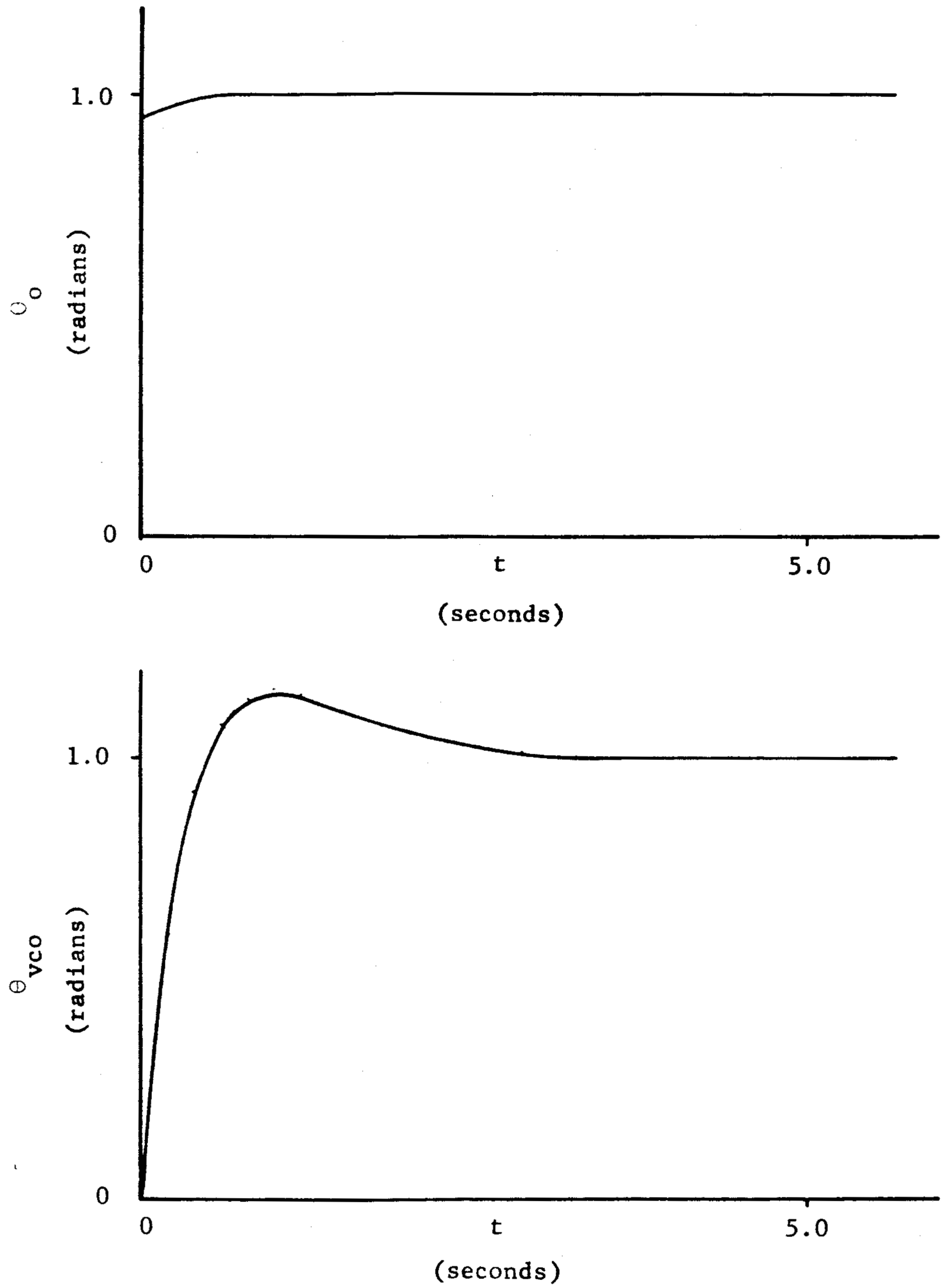


Fig. B-8. Response of Θ_o and Θ_{vco} with τ_1/τ_2=10.0 and K_2/K_1=4.25.

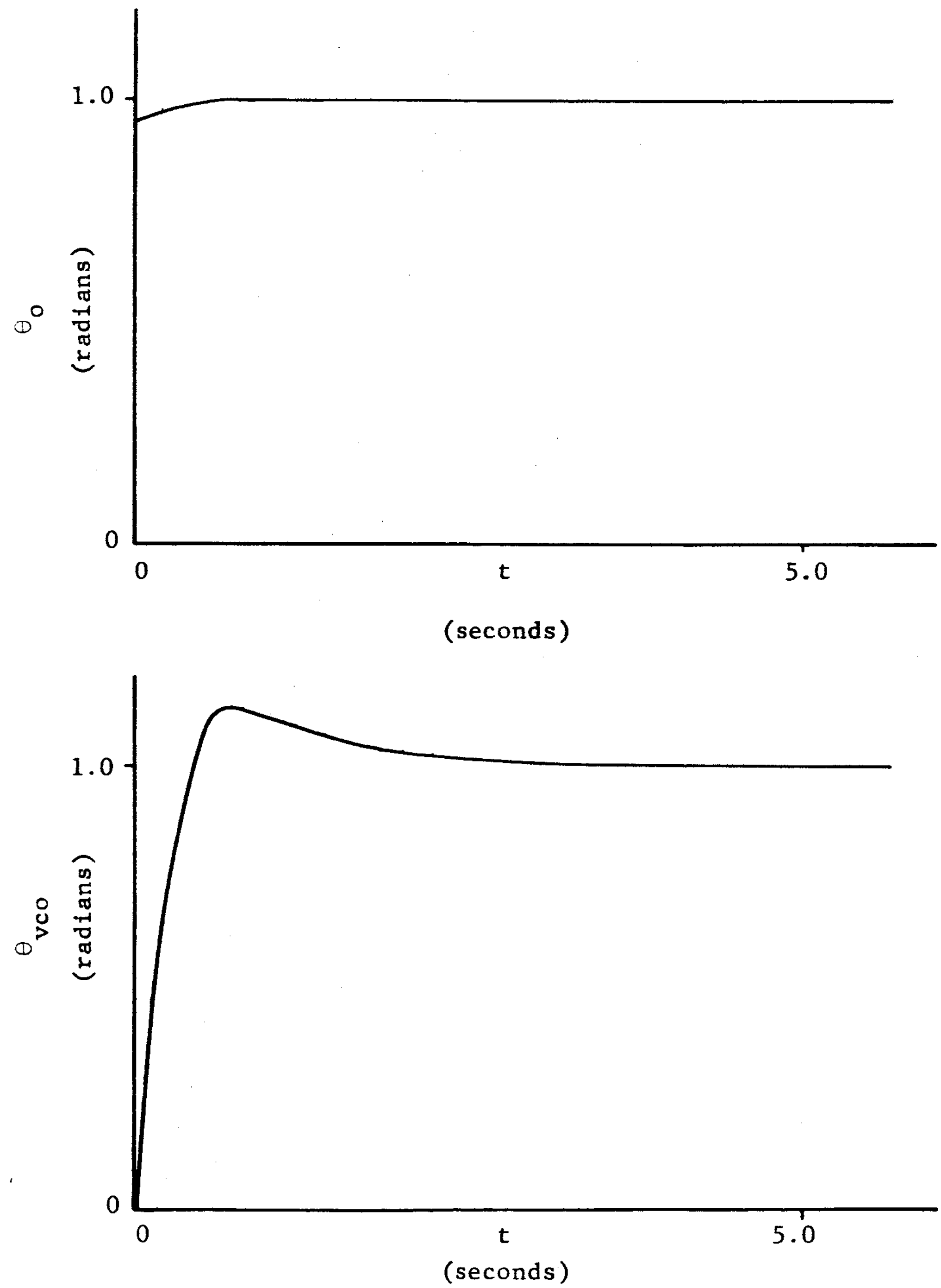


Fig. B-9. Response of Θ_o and Θ_{vco} with τ_1/τ_2=10.0 and K_2/K_1=4.82.